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THE GRAVITY ANOMALIES DERIVED FROM GEOID HEIGHT DATA IN VIETNAM

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Abstract

Both gravity anomaly and geoid height are features of the disturbing potential. Geoid undulations have been determined by gravity anomalies based on the solution of boundary value problem in the potential theory for a long time. Recently, with the broad application of the satellite altimeter data, geoid heights have been determined directly in large regions without gravity anomaly data. With the availability of geoid height data, the opposite direction of calculating gravity anomalies has been considered. This study aims to generate gravity anomaly data in Vietnam based on the least squares collocation approach, of which the correlation between gravity anomalies and geoid heights is the key point. At the same time, in order to simplify the calculation, remove - restore technique was applied. The results not only contributed to making the theory and process more defined, but also verified the determination of gravity anomaly data employing geoid undulations in Vietnam. Additionally, the results are of great significance in constructing a marine gravity model to study sea level rise, and provide information for further exploration and exploitation of marine resources.

Keywords: Gravity anomaly; Geoid height; Least squares collocation; Remove - restore technique.

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1. Introduction

Building a marine gravity database is a very important task because of its great significance to the resources, environment, and climate change. However, measuring sea gravity by the direct method as ship observation is very difficult and expensive. Recently, with the rapid development of science and technology, satellite altimetry is able to determine geoid heights related to gravity anomalies [8, 10] because both the geoid heights and the gravity anomalies are features of the disturbing potential [5]. Therefore, the research on how geoid heights are applied for the identification of gravity anomalies is a modern approach.

The problem of determining gravity anomalies from geoid undulation data is solved by different methods such as the least squares collocation [1], the combination of least squares collocation and vertical deflection [1, 7], and the inverse Vening Meinesz formula [2]. This study employed the least squares collocation to derive gravity anomalies. In addition, for simpler calculations the remove-restore technique described in [1] was applied.

2. The theory of determining gravity anomalies from geoid undulations

The problem of determining gravity anomalies from geoid undulations is mainly solved by the least squares collocation method and the remove - restore technique, of which a priori gravity model is used for removing long wavelengths, the calculation is carried out only for short wavelengths, and then the long wave is restored into the results [1]. The following is a summary of the theory of the method.

The geoid heights and the priori gravitational model are applied for determining residual geoid heights ΔN_1, ΔN_2, …, ΔN_n.

According to [5], we have:

$$N = \frac{T}{\gamma} \tag{1}$$

$$K(\Delta N_i, \Delta N_j) = \left(\frac{T}{\gamma_i}\right)(T_i).\left(\frac{T}{\gamma_j}\right)(T_j)\sum_{l=2}^{\infty}\sigma_l^2\left(\frac{R^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi)$$

$$K(\Delta N_i, \Delta N_j) = \frac{1}{\gamma_i.\gamma_j}\sum_{l=2}^{\infty}\sigma_l^2\left(\frac{R^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi) \tag{2}$$

Where: N is geoid height;

The anomalous gravity potential, *T*, is equal to the difference between the gravity potential *W* and the so-called normal potential *U, T = W-U;*

$P_l(\cos\psi)$ is 1^{th} Legendre polynomial; ψ is the spherical distance from i to j;

r_i and r_j are respectively represent the distance from origin to i and j;

γ is normal gravity;

σ_l^2 is positive constants (degree variance of T):

$$\sigma_l^2 = \left(\frac{GM}{R}\right)^2\left(\sum_{m=0}^{l}c_{lm}^2 + \sum_{m=0}^{l}s_{lm}^2\right) \tag{3}$$

Where G is the gravitational constant, M is the mass of the earth (GM is the product of the mass of the earth and the gravitational constant), R is mean radius of the earth, C_{lm}, S_{lm} are respectively harmonic coefficient of degree l and order m).

Legendre polynomial is as follows:

$$P_l(\cos\psi) = 2^{-l}\sum_{k=0}^{n_1}(-1)^k\frac{(2l-2k)!}{k!(l-k)!(l-2k)!}.(\cos\psi)^{l-2k} \tag{4}$$

In the formula (11), $n_1 = \ell/2$ where ℓ is even number, $n_1 = (\ell-1)/2$ where ℓ is an odd number.

If g represents the gravity at a point on the geoid, and γ represents the normal gravity at this point on the average projection on ellipsoid surface, the difference between g and γ is a gravity anomaly. The residual gravity anomaly Δg at calculated P is as follows [4]:

$$\Delta g_P = K^T_{1\times n}(\Delta N, \Delta g_P).\left[K(\Delta N, \Delta N) + C_\Delta\right]^{-1}_{n\times n}.\Delta\tilde{N}_{n\times 1} \tag{5}$$

where K presents covariance of P and Q, ΔN is the residual geoid height, C_Δ is covariance matrix of error.

The variance function of Δg used is estimated:

$$\sigma^2_{\Delta g} = K_{1\times 1}(\Delta g_P, \Delta g_P) - K^T_{1\times n}(\Delta N, \Delta g_P).\left[K(\Delta N, \Delta N) + C_\Delta\right]^{-1}_{n\times n}.K_{n\times 1}(\Delta N, \Delta g_P) \tag{6}$$

where

$$K^T(\Delta N, \Delta g_P) = \left[K(\Delta N_1, \Delta g_P) \quad K(\Delta N_2, \Delta g_P) \quad \ldots \quad K(\Delta N_n, \Delta g_P)\right] \tag{7}$$

$$K(\Delta N, \Delta N) = \begin{bmatrix} K(\Delta N_1, \Delta N_1) & K(\Delta N_1, \Delta N_2) & \ldots & K(\Delta N_1, \Delta N_n) \\ K(\Delta N_2, \Delta N_1) & K(\Delta N_2, \Delta N_1) & \ldots & K(\Delta N_1, \Delta N_1) \\ \ldots & \ldots & \ldots & \ldots \\ K(\Delta N_n, \Delta N_1) & K(\Delta N_n, \Delta N_2) & \ldots & K(\Delta N_n, \Delta N_n) \end{bmatrix} \tag{8}$$

$$C_\Delta = \begin{bmatrix} c_{11} & c_{12} & \ldots & c_{1n} \\ c_{21} & c_{22} & \ldots & c_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ c_{n1} & c_{n2} & \ldots & c_{nn} \end{bmatrix}; \quad \Delta\tilde{N} = \begin{bmatrix} \Delta\tilde{N}_1 \\ \Delta\tilde{N}_2 \\ \ldots \\ \Delta\tilde{N}_n \end{bmatrix} \tag{9}$$

Covariance of residual geoid undulation K(ΔN_i, ΔN_j) is estimated:

$$K(T_i, T_j) = \sum_{l=2}^{\infty} \sigma_l^2 \left(\frac{R^2}{r_i.r_j}\right)^{l+1} P_l(\cos\psi) \tag{10}$$

The gravity anomaly is detemined by the fundamental formula [6]:

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r}.T \tag{11}$$

Where r represents distance from origin.

Taking the formula (1) and (10) into the formula (11), covariance form between residual geoid height and residual gravity anomaly can be received:

$$K(\Delta g_i, \Delta g_j) = \left(-\frac{\partial}{\partial r_i} - \frac{2}{r_i}\right)(T_i).\left(-\frac{\partial}{\partial r_j} - \frac{2}{r_j}\right)(T_j)\sum_{l=2}^{\infty} \sigma_l^2 \left(\frac{R^2}{r_i.r_j}\right)^{l+1} P_l(\cos\psi)$$

$$K(\Delta g_i, \Delta g_j) = \sum_{l=2}^{\infty} \sigma_l^2 \frac{(l-1)^2}{r_i.r_j}\left(\frac{R^2}{r_i.r_j}\right)^{l+1} P_l(\cos\psi) \tag{12}$$

Taking the formula (10) into the formula (11), we have:

$$K(\Delta g_i,\Delta g_j)=\left(-\frac{\partial}{\partial r_i}-\frac{2}{r_i}\right)(T_i).\left(-\frac{\partial}{\partial r_j}-\frac{2}{r_j}\right)(T_j)\sum_{l=2}^{\infty}\sigma_l^2\left(\frac{R^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi)$$

$$K(\Delta g_i,\Delta g_j)=\sum_{l=2}^{\infty}\sigma_l^2\frac{(l-1)^2}{r_i.r_j}\left(\frac{R^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi) \tag{13}$$

However, determining the covariance functions based on expressions (1), (12) and (13) is not practical because covariances needed for this purpose are just calculated in certain limited degree of N, but only at the approximate level. The remaining order covariances are forced to be modeled.

The covariance function of the Earth's disturbed potential corresponds to one of the most satisfactory models is the following form [4]:

$$K(T_i,T_j)=a\sum_{l=2}^{N}d_l\left(\frac{R^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi)+\sum_{l=N+1}^{\infty}\frac{A}{(l-1)(l-2)(l+b)}\left(\frac{R_B^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi) \tag{14}$$

where a is considered an additional parameter that needs to be determined according to the correlation analysis of empirical data; d_l is the variance of the error of the harmonic coefficients of the gravity potential; B is a number assigned a value of 4, but sometimes, in order to achieve the best asymptote with low order variances, it is possible to obtain 24; A is a constant in units of $(m/c)^4$; R is the average radius of the Earth; R_B is the radius of a sphere that lies fully inside of the Earth.

Covariance function of the residual geoid heights is expressed in following form:

$$K(\Delta N_i,\Delta N_j)=a\sum_{l=2}^{N}d_l\frac{1}{\gamma_i.\gamma_j}\left(\frac{R^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi)+\sum_{l=N+1}^{\infty}\frac{A}{(l-2)(l+b)}\frac{1}{\gamma_i.\gamma_j}\left(\frac{R_B^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi) \tag{15}$$

The cross covariance function between residual geoid heights and residual gravity anomalies:

$$K(\Delta N_i,\Delta g_P)=\frac{a}{\gamma_i}\sum_{l=2}^{N}d_l\frac{(l-1)}{r_P}\left(\frac{R^2}{r_i.r_P}\right)^{l+1}P_l(\cos\psi)+\frac{1}{\gamma_i}\sum_{l=N+1}^{\infty}\frac{A}{(l-2)(l+b)}\frac{1}{r_P}\left(\frac{R_B^2}{r_i.r_P}\right)^{l+1}P_l(\cos\psi) \tag{16}$$

Covariance function of the residual gravity anomaly:

$$K(\Delta g_i,\Delta g_j)=a\sum_{l=2}^{N}d_l\frac{(l-1)^2}{r_i.r_j}\left(\frac{R^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi)+\sum_{l=N+1}^{\infty}\frac{A}{(l-2)(l+b)}\frac{(l-1)}{r_i.r_j}\left(\frac{R_B^2}{r_i.r_j}\right)^{l+1}P_l(\cos\psi) \tag{17}$$

Parameters a, d_l, N, A, and R_B need to be determined according to the correlation analysis of empirical data of the residual geoid heights.

The covariance of anomalous gravity potential is understood as a mean value of all possible products of anomalous gravity potential at two points P and P' which are located at a constant distance on the sphere. Each of these defined covariance values corresponds to a specific spherical distance **ψ**, and that is the values of a function called the covariance function of the disturbing potential. Thus, we will have the definition of covariance function of the anomalous gravity potential as follows:

$$K(\psi_{P,P'}) = \frac{1}{8\pi^2} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} T(P).T(P')d\alpha \sin\theta \, d\theta \, d\lambda, \qquad (18)$$

where α is is the azimuth of PP ', θ và λ are respectively the components of the spherical coordinate of point P, ψ is the distance between P and P'. This function can be expressed as follows:

$$K(P, P') = \sum \sigma_l^2 \left(\frac{R}{r.r'} \right)^{l+1} P_l(\cos\psi), \qquad (19)$$

where:

$$\sigma_l^2 = \left(\frac{GM}{R} \right)^2 \sum_{m=-l}^{l} c_{lm}^2. \qquad (20)$$

The calculation base on the expression (19) is implemented by computing the empirical covariance values corresponding to the different ψ_i.

$$\widehat{K}(\psi_i) = \frac{1}{m_i} \sum_{n=1}^{m_i} [T(P).T(P')]_n, \qquad (21)$$

If the measurement values are the residual geoid undulations, we have [2]:

$$K_{\Delta N}(\psi_i) = \frac{1}{m_i} \sum_{n=1}^{m_i} [\Delta N(P).\Delta N(P')]_n, \qquad (22)$$

where P and P' is point pair where there are known ΔN values, the distance from P to P' satisfies condition:

$$\psi_i - \frac{\Delta\psi}{2} \le \psi \le \psi_i + \frac{\Delta\psi}{2}, \qquad (23)$$

where m_i is the number of point pairs, $\Delta\psi$ is the average range between points at which there is a value ΔN and $\psi_i - \Delta\psi/2 = 0$, if $\psi_i < \Delta\psi/2$.

- The estimation of the covariance function:

After receiving the experimental covariances of the residual geoid heights, it should be approximated by an analytic function of the form (15) with appropriate parameters a, dl, RB, A and N. Then, determining relevant covariance function of the form (16) and (17).

- Computing residual gravity anomalies and restoring the effect of low - frequency components of the gravity field

With known covariance functions, the remaining calculations are to compute residual gravitational anomalies by the expression (17). In order to achieve this goal, it is necessary to solve large standard normal equations with very large numbers corresponding to tens and even hundreds, thousands of values. After receiving the residual geoid height values, the fully geoid undulation values are calculated by the expression:

$$\Delta g = \overline{\Delta} g + \Delta g_{EGM} \qquad (24)$$

where $\overline{\Delta} g$ is residual gravity anomalies, Δg_{EGM} is gravity anomalies that are computed by using (from) Earth gravity as follows:

$$\Delta g_{EGM} = \frac{GM}{r^2}\left[\sum_{n=2}^{N}\left(\frac{a}{r}\right)^n (n-1)\sum_{m=0}^{n}\left(c_{n,m}\cos m\lambda + s_{n,m}\sin m\lambda\right)\bar{P}_{n,m}(\sin\varphi)\right] \quad (24)$$

3. Experiment

3.1. Study area and data

The study was carried out in the area defined by $8^0 < \varphi < 22^0$, $108^0 < \lambda < 115^0$, where φ is the latitude and λ is the longitude. Set of 4028 points, which having geoid heights identified from the altimeter data, are evenly distributed in the experimental area.

To apply the remove - restore technique, the EGM96 global gravity model was chosen as a priori model. Information about the model is presented in [3].

3.2. Technical procedures

Data processing includes:

- Firstly, determining residual geoid heights: employing geoid heights minus the corresponding EGM96 geoid model values;

- Secondly, calculating empirical covariances and parameters of analytic covariance functions of residual geoid heights;

-Thirdly, determining residual gravity anomalies: Based on the determination of the appropriate parameters of the residual geoid heights' covariance function, residual gravimetric anomalies were pinpointed according to the formulas described in Section 2.

- Finally, computing gravity anomalies: restoring the corresponding EGM96 gravity anomaly model values.

The fully technical procedures are shown in Fig. 1.

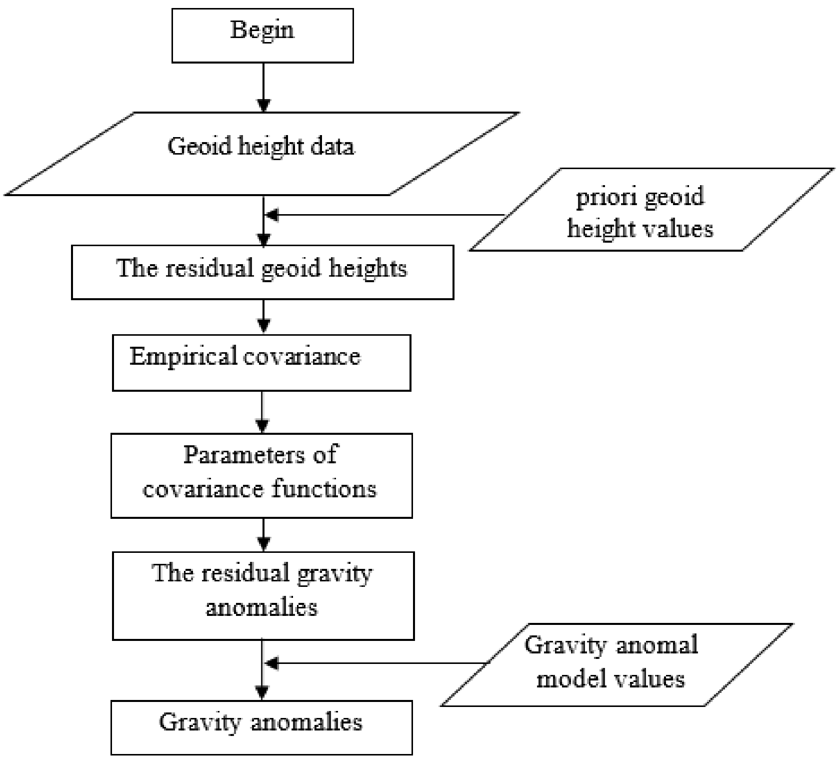

Figure 1: The flowchart of gravity anomaly determination

Calculations were performed by using the GRAVSOFT package [6, 9].

3.3. Results and Analysis

- The determination of the residual geoid heights

Making use of the formula, the residual geoid heights were calculated. The data extracted from the results are given in Tab. 1.

Table 1. The residual geoid heights in study area

N_0	Latidude (^{0})	Longtitude (^{0})	Height (m)	Residual geoid height (m)
1	8.0057	108.265	0	1.374
2	8.0090	111.858	0	1.205
3	8.0094	104.672	0	1.721
4	8.0146	110.421	0	1.225
5	8.0192	103.273	0	1.581
6	8.0197	108.303	0	1.375
7	8.0217	106.831	0	1.29
8	8.0217	103.237	0	1.577
....	....	....	....	....
4021	22.1128	113.660	0	1.251
4022	22.1464	113.696	0	1.376
4023	22.2123	113.712	0	1.219
4024	22.2782	113.729	0	1.176
4025	22.3106	113.611	0	1.404
4026	22.3442	113.745	0	1.182
4027	22.4101	113.761	0	1.251
4028	22.4760	113.777	0	1.339

- The determination of empirical covariances and parameters of analytic covariance functions of the residual geoid heights

In this study, the average spherical distance $\Delta\psi$ was taken at 10'. The empirical covariance values of the residual geoid heights were computed for the spherical distance ψ ranging from 0^0 up to 3^0.

After receiving the empirical covariance values of the residual geoid undulations, the determination of parameters of analytic covariance functions was carried out. The results were as follows: N = 120, a = 0.2202, R_B - R = - 977.52m, A= 45018 $(m/s)^4$, and the variance of gravity anomalies is 32.28 $mgal^2$. The empirical and analytical covariance values of residual geoid heights are shown in Tab. 2.

Table 2. The empirical and analytic covariance values of the residual geoid heights

N_0	Distance (^{0})	Empirical covariance values, (m^2)	Analytic covariance values, (m^2)
1	0.000	0.0294	0.0294
2	0.167	0.0268	0.0274
3	0.333	0.0232	0.0234

N_0	Distance (^{0})	Empirical covariance values, (m^2)	Analytic covariance values, (m^2)
4	0.500	0.0189	0.0189
5	0.667	0.0129	0.0145
6	0.833	0.0109	0.0105
7	1.000	0.0099	0.0072
8	1.167	0.0080	0.0046
9	1.333	0.0044	0.0027
10	1.500	0.0047	0.0014
11	1.667	0.0040	0.0006
12	1.833	0.0040	0.0001
13	2.000	0.0020	0.0000
14	2.167	0.0027	0.0000
15	2.333	0.0030	0.0001
16	2.500	0.0030	0.0002
17	2.667	0.0022	0.0003
18	2.833	0.0015	0.0003
19	3.000	0.0014	0.0002

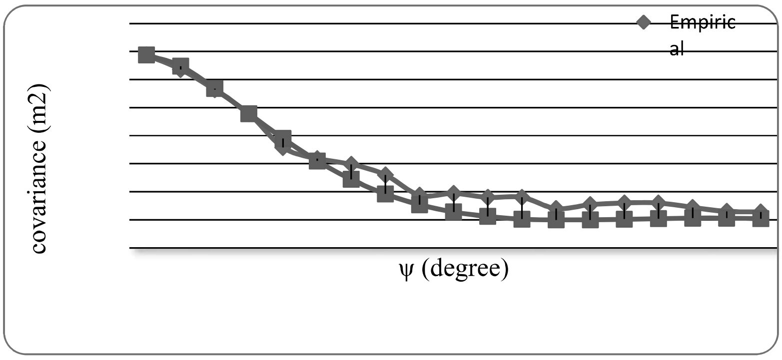

Figure 2: The graph of empirical and analytical residual geoid height covariance

- The identification of residual gravity anomalies and restoring the corresponding EGM96 gravity anomaly model values

The cross-covariances between residual gravimetric anomalies and residual geoid undulations and auto-covariances of gravimetric anomalies were identified by using the parameters of the analytic covariance function of residual geoid heights. Then, residual gravity anomalies were determined, and the corresponding EGM96 gravity anomaly model values were restored. The calculated gravity anomaly map generated with the Surfer software is shown in Fig. 3.

The results show that the gravity anomalies vary remarkably in the study area. The highest and lowest values are 17.5 mgal and -13.5 mgal respectively. The most common values of the gravity anomalies range from -2.0 up to 2.0 mgal, and account for approximately 70% of the total gravity anomalies.

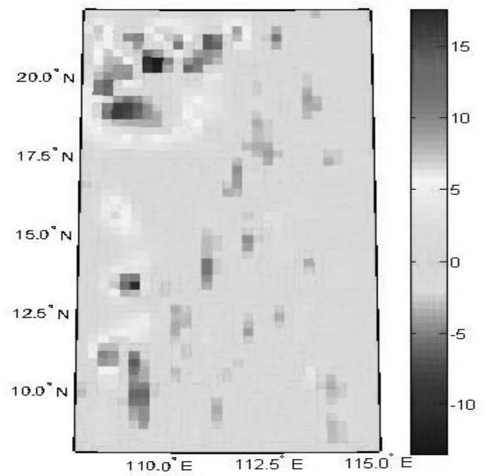

Figure 3: The gravity anomaly map of the study area

4. Conclusion

Using the theory of the least squares collocation and the remove-restore technique, the paper has determined gravity anomalies from geoid undulation data in the experimental area based on the correlation between the two quantities. The results not only contribute to clarifying the theoretical foundation and the technical procedure, but also confirming the feasibility of the determining gravity data based upon geoid heights in Vietnam. However, this research direction should be followed up with other methods such as the combination of least squares collocation and vertical deflection and the inverse Vening Meinesz formula. In addition, the comparison and assessment of the accuracy of the methods should be taken into consideration.

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